

Investigating the Dynamic Volatility, and Conditional Covariance of Stock Market Indicators in Nigeria using VEC–AI–BEKK Approach

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Abstract: This research explores the characteristics of distribution, interactions of dynamic volatility, and the conditional covariance framework connecting the All-Share Index (ASI) and Market Capitalization (MC) within the Nigerian capital market, utilizing a hybrid multivariate approach comprising VEC–LSTM–BEKK, VEC–ANN–BEKK, and VEC–SVR–BEKK. Using Python modules (arch, statsmodels, TensorFlow, PyTorch), the research uses 1,799 monthly secondary data points from the Central Bank of Nigeria from January 1996 to June 2024. Preliminary Jarque-Bera tests show non-normality, and the Augmented Dickey-Fuller and Phillips-Perron tests corroborate return stationarity. With low average returns and substantial standard deviations (0.1599 and 0.3839, respectively), ASI (0.0093) and MC (0.0169) demonstrate market capitalization volatility. Both return series display high kurtosis ("ASI"=120.393; "MC"=164.085) and positive skewness ("ASI"=0.7037; "MC"=0.2928). Jarque-Bera tests ("ASI"=207,789.271; "MC"=385,955.456; p=0.000) disprove Unit root tests (p=0.000) and KPSS tests (p=0.100) prove stationarity. Basic raw return correlation is -0.14. MC is more volatile than ASI, with conditional standard deviations of 1.1014 to 1.4059 versus 1.0605 to 1.3075, according to the initial conditional covariance matrices in all three models. VEC–LSTM–BEKK yields the lowest baseline conditional variances (1.1247 for ASI; 1.2130 for MC), middle (1.5908; 1.9225), and highest risk estimates (1.7095; 1.9765). All models had negative off-diagonal covariance factors (-0.2847 to -0.3460), indicating an immediate inverse co-volatility dynamic between aggregate market value and stock index movements.

Keywords: All-Share Index (ASI); Phillips-Perron Tests; Python Libraries; Optimizing Portfolios; Non-Normal Distribution; Managing Equity Risk; Market Capitalization (MC).

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1. Introduction

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The erratic behaviour of financial markets, especially in emerging markets such as Nigeria, is marked by volatility clustering, structural instability, asymmetric reactions to shocks, and complex interrelationships among key market indicators. Financial variables are rarely independent, as changes in interest rates, exchange rates, inflation, money supply, commodity prices, investor sentiment and stock market conditions may influence each other simultaneously [5]. The Nigerian financial market is particularly susceptible to these interactions, given its sensitivity to domestic macroeconomic developments and external pressures from global financial conditions, commodity price fluctuations, international capital flows and geopolitical uncertainties [7]. Empirical observations show that the Nigerian stock market has been characterised by persistent volatility, responses to economic shocks and dynamic risk structures. Such features imply that static or merely linear assumptions cannot adequately represent financial relations [8]. Instead, robust econometric frameworks are needed to identify long-term equilibrium relationships while capturing short-term adjustments, nonlinear dependencies, volatility persistence and changing conditional covariances among financial variables. Financial market volatility is not normally distributed over time [10]. Volatility clustering arises because periods of relatively low market uncertainty are often followed by periods of significant fluctuations.

This phenomenon could be more pronounced in an emerging market such as Nigeria, as financial markets can respond strongly to changes in monetary policy, political events, exchange-rate pressures, inflationary conditions, oil-price swings, and global economic shocks. Moreover, the economic environment in which the policy is implemented can affect the magnitude and direction of the response [11]. The effect of a positive shock in a period of economic expansion may be different than the effect of the same shock in a period of recession or financial distress. Therefore, models that assume constant relationships and homogeneous adjustment processes may not fully capture the actual behaviour of Nigerian financial markets [14]. These features underscore the need for methodologies flexible enough to identify both equilibrium-type relations and time-varying market dynamics [12]. Traditionally, the Vector Error Correction Model (VECM) has been widely used to study long-run relationships among integrated financial and macroeconomic variables in Nigeria. The main advantage of the VECM is that it can account for short-run dynamics and long-run equilibrium information when a group of non-stationary variables is cointegrated [13]. The VECM provides a unified framework for identifying deviations from long-term equilibrium and assessing the subsequent adjustment process, rather than examining each financial indicator in isolation.

This feature has been particularly useful for studying the relations between stock market development, macroeconomic conditions and other financial indicators. For instance, studies investigating the relationship between stock market growth and macroeconomic factors have employed VECM to identify equilibrium relationships between market indicators. These models are particularly relevant in the Nigerian context because financial variables may share common stochastic trends, even though their short-run dynamics may differ markedly. The traditional VECM, while useful, has serious methodological limitations in light of increasingly complex financial environments. The VEC model is useful for detecting cointegration and estimating the speed of adjustment to long-term equilibrium. Still, the model's standard form is largely linear, and it assumes the adjustment mechanism is stable over the study period. However, financial markets often exhibit nonlinear and regime-dependent behaviour [15]. In stressed markets, the movement toward equilibrium can be rapid, whereas in stable markets it can be considerably slower. Further, the correlation between financial indicators could change after significant economic or institutional shocks. Such properties pose challenges to a model that relies on a relatively stable linear structure. Thus, although the VECM can capture long-term relationships, it does not fully capture the dynamics of these relationships across different volatility regimes or the effect of nonlinear interactions on shock transmission to the financial system [6].

Another important limitation is how it treats volatility and covariance. Traditional VECM specifications focus on the conditional mean and variables' adjustment to equilibrium. Yet, financial risk is not only a function of expected returns or mean relations [7]. Conditional variances of financial variables and covariances between markets may vary substantially over time. In times of uncertainty, increases in the volatility of one financial indicator may be transmitted to another, leading to volatility spillovers and stronger cross-market dependence. These effects are especially important in emerging financial markets, where even modest domestic or international shocks can trigger significant changes in investor behaviour [8]. Hence, the methodological approach that focuses only on mean-level cointegration may neglect relevant information in the conditional variance-covariance structure. This limitation prevents the conventional VECM from fully capturing dynamic instability, regime changes, nonlinear interactions and volatility transmission often observed in financial markets [9]. With these limitations in mind, recent methodological advances have increasingly explored combining Artificial Intelligence (AI) and machine learning with traditional econometric approaches. AI-based techniques can identify complex nonlinear relationships that traditional econometric models may not explicitly specify [10]. Machine learning algorithms can learn patterns from historical observations and capture complex interactions among predictors, whereas conventional linear models cannot. This is particularly helpful for financial forecasting, where relationships between variables can be non-linear, unstable and influenced by multiple factors.

Thus, AI methods can complement traditional econometric methods by increasing predictive flexibility while preserving the economic information provided by long-term equilibrium analysis [11]. However, empirical studies in the Nigerian financial

market that simultaneously combine cointegration-based modelling, AI learning techniques and multivariate volatility analysis remain scarce. Prior work has focused on separate parts of the problem rather than an integrated framework that can handle equilibrium relationships, nonlinear dependencies and changing volatilities. Previous attempts to improve volatility modelling through extensions such as VAR-MGARCH, BEKK-MGARCH, and Dynamic Conditional Correlation (DCC) models have greatly enhanced understanding of volatility transmission and conditional dependence [12]. These approaches improve on constant-variance models because they allow volatility and covariance to change over time. However, even sophisticated MGARCH specifications can run into problems when financial relationships are highly nonlinear or when the underlying data-generating process changes dynamically. Thus, integrating adaptive AI-based learning into such frameworks may provide an additional mechanism for identifying patterns that conventional parametric specifications may not capture [13]. The BEKK-MGARCH framework is particularly relevant, as it provides a systematic way to model the time-varying conditional variances and covariances of multiple financial variables. Multivariate GARCH models explicitly model interactions between conditional variances and covariances, whereas univariate volatility models examine each series separately. Moreover, the BEKK specification ensures that the conditional covariance matrix is positive definite under suitable parameter restrictions. The BEKK model helps analyse interrelated financial markets.

This capability matters in the Nigerian stock market because risk in one indicator may affect the conditional risk of another. Changes in market volatility may therefore spill over into portfolio allocation, investment decisions, risk management, and financial stability. However, the BEKK framework still relies on a specified functional structure [14]. It may not fully capture the highly nonlinear relationships or intricate patterns that AI-based approaches can learn. Hence, combining VEC, AI, and BEKK provides a potentially more holistic framework for understanding the Nigerian financial market. The VEC component can determine whether a long-term equilibrium relationship exists among the main financial indicators and measure the rate at which short-term deviations correct [1]. The AI component then provides a framework with nonlinear learning capabilities to detect complex relationships and improve predictive performance. The BEKK component can also capture the time-varying nature of volatility and conditional covariance, thereby providing information on the transmission of risk among the selected financial indicators. The proposed framework does not view these approaches as competing alternatives, but as complementary methodological components that capture different dimensions of financial-market behaviour [2]. The proposed VEC–AI–BEKK framework thus aims to overcome limitations of conventional linear econometric models and restricted MGARCH specifications. The main message is that long-run equilibrium relations, nonlinear dependence and dynamic volatility should be considered simultaneously, not separately. This integration can deepen understanding of interactions among financial indicators across time horizons.

Long-run relationships may reflect fundamental equilibrium tendencies, while short-run error-correction mechanisms may account for transitory deviations. At the same time, AI-based methods can detect nonlinear patterns in the data, while BEKK-MGARCH can model the evolution of conditional variance and covariance [3]. The resulting combined framework is thus expected to provide a more flexible representation of financial market dynamics than approaches based solely on linear cointegration or classical volatility modelling. The Nigerian context is especially pertinent for investigating this integrated approach. Nigeria is an emerging economy, and its financial market is influenced by domestic economic policies and structural developments, as well as by international financial conditions. Changes in world commodity markets, especially oil revenues, can have far-reaching effects on exchange rates, government revenues, inflation, investor confidence and stock market performance [4]. Domestic monetary and fiscal policies might also impact liquidity conditions and asset prices. These interactions can lead to periods when financial variables move closely together and other periods when their relationships weaken or change direction [5]. Thus, a model that captures evolving dependencies can provide more meaningful information to investors, policymakers, financial institutions, and researchers than one based on fixed relationships. Specifically, the study seeks to: (i) examine the long-run cointegration relationships among the major stock market indices in Nigeria; (ii) construct a dynamic volatility and conditional covariance framework using the BEKK-MGARCH approach; (iii) utilise AI-driven techniques to capture nonlinear relationships and improve predictive accuracy; and (iv) evaluate the performance of the hybrid VEC–AI–BEKK model against traditional econometric and volatility-modeling techniques [6].

The first aim is to determine the basic long-term relationships between the selected financial indicators and to establish the presence of common equilibrium tendencies. The second objective is to analyse the dynamic risk structure. It investigates the evolution of conditional variances and covariances over time and the existence of volatility transmission among the indicators. The third aim is to use AI-based learning methods to capture non-linear dependencies that standard econometric specifications may overlook [7]. The fourth objective is therefore to provide an empirical basis for assessing whether the integrated framework delivers meaningful improvements in explanatory and predictive performance relative to well-established approaches. Using this integrated methodology, the study aims to contribute to the growing literature on advanced financial econometrics and intelligent financial modelling. The contribution is not only the combination of different statistical techniques. The proposed framework addresses different but related features of financial data. Cointegration indicates long-run equilibrium, AI provides nonlinear learning and predictive flexibility, and BEKK-MGARCH provides a mechanism for analysing dynamic conditional volatility and covariance [8]. Taken together, these components may offer a more complete representation of financial

interconnectedness in an emerging market context. Moreover, the proposed approach may have practical implications for financial risk management and investment decision-making. Better identification of the long-term relationships can help investors to understand the fundamental links between financial indicators. Accurate modelling of volatility and covariance can help with portfolio diversification and risk assessment [9].

AI's non-linear predictive abilities can provide an added advantage in detecting market patterns that traditional econometric tools struggle to capture. For policymakers and financial regulators, understanding volatility spillovers and changing interdependencies can provide useful evidence for monitoring systemic risk and designing appropriate policy responses [10]. Hence, the proposed framework is relevant from academic financial modelling to real-world applications in investment management, market surveillance and financial stability. Overall, the shifting structure of financial markets demands analytical frameworks that go beyond the assumption of stable linear relationships. The Nigerian stock market is a special case as its dynamics reflect the interaction of domestic economic conditions, external shocks, investor expectations and changing risk perceptions [11]. VECM is a useful benchmark to identify long-run equilibrium relations. However, the traditional linear VECM framework limits its ability to model nonlinear dependencies and changing volatility. Similarly, MGARCH and BEKK models offer advanced methods for modeling conditional variances and covariances; however, incorporating adaptive AI-based learning can improve their performance. Thus, the VEC-AI-BEKK framework proposed in this study offers an in-depth approach to investigating long-run equilibrium, nonlinear interactions, dynamic volatility, and conditional covariance within a single analytical framework [12]. The study empirically tests this framework against conventional approaches, providing further evidence for modelling and forecasting interlinked financial variables and contributing a more flexible and thorough approach to volatility and risk analysis in emerging financial markets such as Nigeria [14].

2. Methodology

The quantitative analytical procedure begins with collecting secondary monthly returns, volatility, and macroeconomic indicators from the Central Bank of Nigeria website spanning January 1996 to June 2024, yielding 1,799 data points. Data preprocessing and model execution are carried out in Python using specialized econometric and machine learning libraries, including arch, statsmodels, numpy, scipy, scikit-learn, TensorFlow, and PyTorch. Initial transformations convert the raw price series into continuously compounded logarithmic returns to establish stationarity and control for heteroskedasticity. The return $R_{1,t}$ for the All-Share Index (ASI) and $R_{2,t}$ for Market Capitalization (MC) at time t are derived from current index values $P_{1,t}$ and $P_{2,t}$ relative to previous period values $P_{1,t-1}$ and $P_{2,t-1}$ using the logarithmic equations:

$$R_{1,t} = \ln\left(\frac{P_{1,t}}{P_{1,t-1}}\right) \quad (1)$$

$$R_{2,t} = \ln\left(\frac{P_{2,t}}{P_{2,t-1}}\right) \quad (2)$$

Time series plots are first generated to visually inspect the data for underlying long-term trends, structural breaks, and seasonal patterns. Descriptive statistics and normality checks are then executed using the Jarque-Bera (JB) test statistics, defined as:

$$JB = \frac{N}{6} \left(S^2 + \frac{(K-3)^2}{4} \right) \quad (3)$$

Where S represents skewness, K denotes kurtosis, and N is the sample size. Departures from normality motivate the use of multivariate GARCH models under non-normal error distributions. Interrelationships between variables are evaluated using the sample correlation coefficient. r_{xy} :

$$r_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}} \quad (4)$$

Stationarity and integration orders are formally tested using Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. The parameter δ is evaluated across three structural specifications:

$$\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^p \beta_i \Delta Y_{t-i} + \varepsilon_t \quad (5)$$

$$\Delta Y_t = \alpha_0 + \delta Y_{t-1} + \sum_{i=1}^p \beta_i \Delta Y_{t-i} + \varepsilon_t \quad (6)$$

$$\Delta Y_t = \alpha_0 + \delta Y_{t-1} + \alpha_2 t + \sum_{i=1}^p \beta_i \Delta Y_{t-i} + \varepsilon_t \quad (7)$$

Where the null hypothesis $H_0: \delta = 0$ (indicating a unit root) is tested against the alternative $H_1: \delta < 0$. If calculated t -statistics exceed MacKinnon critical values at the 5% significance level, the series is declared stationary; otherwise, first differencing is applied [2]. For the Hybrid VEC–AI–BEKK Model Architecture, integrating Vector Error Correction (VEC), advanced nonlinear mean estimators (LSTM, ANN, SVR), and the BEKK-GARCH framework yields a hybrid Mean-Variance Decomposition Architecture. This hybrid methodology captures three structural layers in the Nigerian Capital Market (NCM): Cointegration and Long-run Equilibrium via the VECM cointegrating vector (r), non-linear residual forecasting via machine learning algorithms ($ML \in \{LSTM, ANN, SVR\}$) and multivariate dynamic volatility and spillover via the BEKK parameterization. Let Y_t be a (2×1) vector of log-returns in the Nigerian capital market at time t :

$$Y_t = \begin{bmatrix} R_{ASI,t} \\ R_{MCAP,t} \end{bmatrix} = \begin{bmatrix} \ln(ASI_t/ASI_{t-1}) \\ \ln(MCAP_t/MCAP_{t-1}) \end{bmatrix} \quad (8)$$

Where, $R_{ASI,t}$ is the All-Share Index (ASI) return and $R_{MCAP,t}$ is the Market Capitalization (MCAP) return. For the Linear Cointegration Base (VEC Model), if the price series:

$$P_t = [\ln(ASI_t), \ln(MCAP_t)]^T \quad (9)$$

Are non-stationary at levels $I(1)$ but cointegrated $CI(1,1)$, the linear VEC (p) specification is:

$$\Delta P_t = \mu + \Pi P_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta P_{t-i} + \epsilon_t \quad (10)$$

Where: $\Pi = \alpha\beta^T$, with β representing the long-run cointegrating vector, and α representing the speed-of-adjustment matrix. $ECT_{t-1} = \beta^T P_{t-1}$ is the Error Correction Term. The linear mean return prediction is $\hat{\mu}_{VEC,t} = E[\Delta P_t | \mathcal{F}_{t-1}]$:

$$\hat{\mu}_{VEC,t} = \mu + \alpha ECT_{t-1} + \sum_{i=1}^{p-1} \Gamma_i Y_{t-i} \quad (11)$$

The linear residual vector is extracted as:

$$e_t = Y_t - \hat{\mu}_{VEC,t} \quad (12)$$

2.1. Non-Linear Residual Mean Modeling

To model high-frequency non-linear dynamics, the residual e_t is fitted using one of three machine learning algorithms (f_{ML}). The final conditional mean return estimate is parameterised as:

$$Y_t = \hat{\mu}_{VEC,t} + f_{ML}(X_t) + \eta_t \quad (13)$$

Where $X_t = [e_{t-1}, \dots, e_{t-k}, Y_{t-1}, \dots, Y_{t-k}]$ represents the feature matrix of lagged residuals and returns.

2.2. VEC–LSTM–BEKK Formulation

The LSTM cell models temporal dependencies in e_t through gate mechanics:

2.2.1. LSTM Equations

- **Forget Gate:** $f_t = \sigma(W_f \cdot [h_{t-1}, X_t] + b_f)$
- **Input Gate:** $i_t = \sigma(W_i \cdot [h_{t-1}, X_t] + b_i)$
- **Candidate Cell State:** $\tilde{C}_t = \tanh(W_c \cdot [h_{t-1}, X_t] + b_c)$
- **Cell State Update:** $C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$
- **Output Gate:** $o_t = \sigma(W_o \cdot [h_{t-1}, X_t] + b_o)$
- **Hidden State:** $h_t = o_t \odot \tanh(C_t)$
- **VEC-LSTM Mean Vector:** $\hat{Y}_t^{\text{VEC-LSTM}} = \hat{\mu}_{VEC,t} + W_y h_t + b_y$

2.3. VEC–ANN–BEKK Formulation

A Multi-Layer Perceptron (MLP) mapping function with L hidden layers:

2.3.1. ANN Equations

$$a^{(1)} = g^{(1)}(W^{(1)}X_t + b^{(1)}) \quad (14)$$

$$a^{(l)} = g^{(l)}(W^{(l)}a^{(l-1)} + b^{(l)}) \quad \text{for } l = 2, \dots, L \quad (15)$$

Where $g(\cdot)$ is an activation function such as ReLU or tanh:

- **VEC-ANN Mean Vector:** $\hat{Y}_t^{\text{VEC-ANN}} = \hat{\mu}_{VEC,t} + W^{(L+1)}a^{(L)} + b^{(L+1)}$

2.4. VEC-SVR-BEKK Formulation

A Support Vector Regression (SVR) using a kernel trick $\phi(X_t)$ (e.g., Radial Basis Function $K(x_i, x_j) = \exp(-\gamma\|x_i - x_j\|^2)$) solves the optimization problem:

$$\min_{w,b,\xi,\xi^*} \frac{1}{2} \|w\|^2 + C \sum_{i=1}^N (\xi_i + \xi_i^*) \quad (16)$$

Subject to:

$$\begin{cases} e_i - (w^T \phi(X_i) + b) \leq \epsilon + \xi_i \\ (w^T \phi(X_i) + b) - e_i \leq \epsilon + \xi_i^* \\ \xi_i, \xi_i^* \geq 0 \end{cases} \quad (17)$$

2.4.1. VEC-SVR Mean Vector

$$\hat{Y}_t^{\text{VEC-SVR}} = \hat{\mu}_{VEC,t} + \sum_{i=1}^N (\alpha_i - \alpha_i^*) K(X_i, X_t) + b_{svr} \quad (18)$$

For the multivariate volatility structure (BEKK-GARCH Integration), the residual error vector from the hybrid mean model is defined as:

$$\eta_t = Y_t - \hat{Y}_t^{\text{VEC-ML}} \quad \text{where ML} \in \{\text{LSTM, ANN, SVR}\} \quad (19)$$

Researchers model the conditional error vector as:

$$\eta_t = H_t^{1/2} z_t, \quad z_t \sim \text{i.i.d. } (0, I_2) \quad (20)$$

Where H_t is the (2×2) conditional covariance matrix between ASI and MCAP returns:

$$H_t = \begin{bmatrix} h_{ASI,t}^2 & h_{ASI,MCAP,t} \\ h_{MCAP,ASI,t} & h_{MCAP,t}^2 \end{bmatrix} \quad (21)$$

2.5. BEKK (1, 1, K) Parameterization

To guarantee that H_t is positive definite for all t , the BEKK (1,1) specification is defined as:

$$H_t = C^* C^{*T} + \sum_{k=1}^K A_k^T \eta_{t-1} \eta_{t-1}^T A_k + \sum_{k=1}^K B_k^T H_{t-1} B_k \quad (22)$$

For a bivariate system ($K = 1$), the structural matrices are expanded as:

$$C^* = \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \quad (23)$$

- Matrix A captures short-run ARCH volatility shocks and cross-market spillovers.
- Matrix B captures long-run GARCH persistence and cross-market persistence.

2.6. Component-Level Matrix Expansion

Expanding the diagonal and off-diagonal covariance equations yields the individual volatility and co-volatility equations:

- **ASI Conditional Variance ($h_{ASI,t}^2$):** $h_{11,t} = c_{11}^2 + (a_{11}\eta_{1,t-1} + a_{21}\eta_{2,t-1})^2 + (b_{11}h_{11,t-1}^{1/2} + b_{21}h_{12,t-1}^{1/2})^2$
- **MCAP Conditional Variance ($h_{MCAP,t}^2$):** $h_{22,t} = (c_{21}^2 + c_{22}^2) + (a_{12}\eta_{1,t-1} + a_{22}\eta_{2,t-1})^2 + (b_{12}h_{12,t-1}^{1/2} + b_{22}h_{22,t-1}^{1/2})^2$
- **Cross-Market Covariance ($h_{ASI,MCAP,t}$):** $h_{12,t} = c_{11}c_{21} + (a_{11}\eta_{1,t-1} + a_{21}\eta_{2,t-1})(a_{12}\eta_{1,t-1} + a_{22}\eta_{2,t-1}) + (b_{11}h_{11,t-1}^{1/2} + b_{21}h_{12,t-1}^{1/2})(b_{12}h_{12,t-1}^{1/2} + b_{22}h_{22,t-1}^{1/2})$

The combined parameters $\Theta = \{\mu, \alpha, \beta, \Gamma, W_{ML}, C^*, A, B\}$ are jointly estimated by maximizing the Quasi-Maximum Likelihood Estimation (QMLE) objective function under a multivariate Student's t or Gaussian distribution assumption:

$$\mathcal{L}(\Theta) = -\frac{TN}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T (\ln |H_t(\Theta)| + \eta_t(\Theta)^T H_t(\Theta)^{-1} \eta_t(\Theta)) \quad (24)$$

Where $N = 2$ and T is the number of trading periods evaluated in the Nigerian Stock Market dataset.

3. Result

Figure 1 illustrates the timeline of the All-Share Index (ASI), while Figure 2 depicts trends in market capitalization (MC) in the Nigerian capital market over the study period. These visuals offer initial insight into the Nigerian capital market's progression, growth characteristics, structural shifts, and potential volatility. Both the All-Share Index and market capitalization show an upward trajectory over time, indicating growth in the Nigerian equity market, rising valuations and shifting investor expectations.

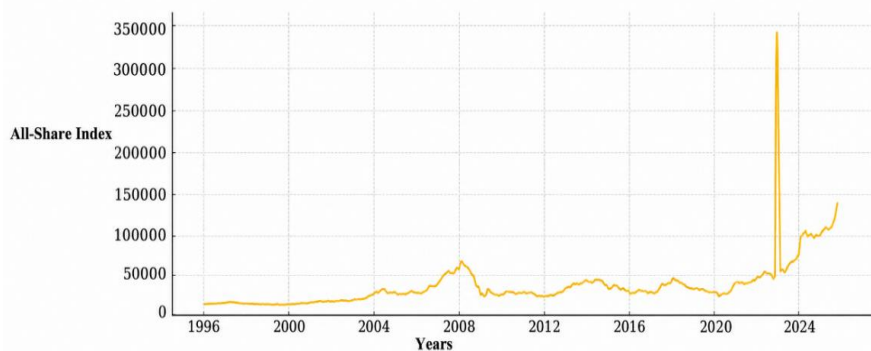


Figure 1: Time plot of the all-share index

Table 1 indicates that the average returns for both ASI and Market Capitalization (MC) are low, with means of 0.0093 and 0.0169, respectively. Their respective medians, 0.0038 and 0.0144, suggest a slight right-tail effect in both distributions.

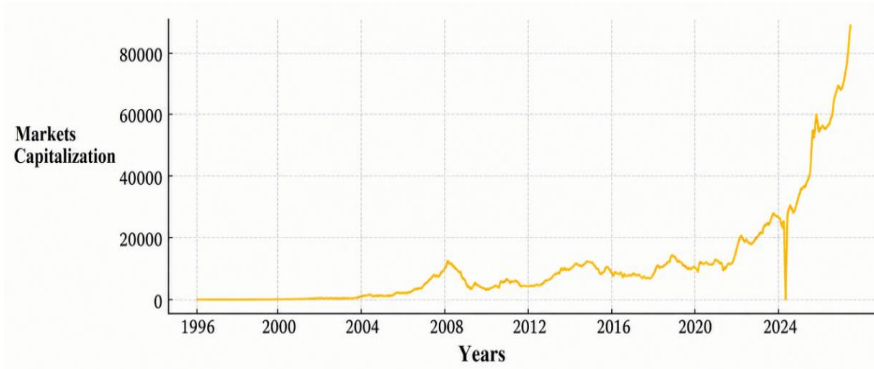


Figure 2: Time plot of market capitalizations

The standard deviations are 0.1599 for ASI and 0.3839 for MC, reflecting significant fluctuations, with MC returns exhibiting greater variation. Both datasets exhibit positive skew (ASI = 0.7037; MC = 0.2928), indicating that occasional substantial positive surprises skew the return distributions. The Kurtosis Figures are remarkably high, at 120.393 for ASI and 164.085 for MC, indicating pronounced leptokurtosis and extremely heavy tails compared to a standard distribution. The Jarque–Bera test statistics are 207,789.271 for ASI and 385,955.456 for MC, both with p-values of 0.000, strongly rejecting the assumption of normality at the 1% significance level. In summary, the findings confirm that returns in the Nigerian capital market exhibit high volatility, departures from normality, and significant tail risk, supporting the use of nonlinear volatility models such as MGARCH.

Table 1: Descriptive statistics of the raw and returns on price in Nigerian capital markets

Series	Mean	Median	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	JB p-Value
ASI Returns	0.0093	0.0038	0.1599	0.7037	120.393	207789.271	0.000
MC Returns	0.0169	0.0144	0.3839	0.2928	164.085	385955.456	0.000

Figure 3 illustrates the relationship between the original series and the return series of variables from the Nigerian capital market. The correlation coefficient between market capitalization and the All-Share Index is -0.14 , indicating a weak negative association. Despite this negative relationship, its small magnitude suggests that short-term fluctuations in market capitalization do not align significantly with ASI returns.

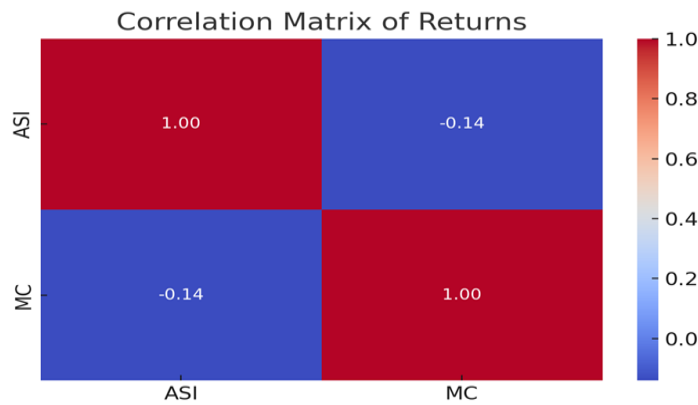


Figure 3: Plot for correlation matrix between raw and returns on Nigerian capital markets

This minimal correlation suggests that relying solely on a single immediate linear relationship might not fully explain the interplay between the two variables. Table 2 provides compelling evidence of stationarity for both ASI and MC returns, as assessed by the ADF and Phillips-Perron tests, where both variables exhibit significantly large negative test values of approximately -18.788 (ASI) and -18.787 (MC), respectively, alongside p-values of 0.000. This allows us to reject the unit root hypothesis at the 5% significance level, suggesting that both datasets are stationary under these parametric and nonparametric tests.

Table 2: Unit root test results for ASI and MC returns on Nigerian capital markets

Variable	Test	Test Statistic	p-Value	Decision ($\alpha = 0.05$)	Conclusion
ASI_ret	Augmented Dickey–Fuller (ADF)	-18.788	0.000	Reject H_0	Stationary
	Phillips–Perron (PP)	-18.789	0.000	Reject H_0	Stationary
	KPSS	0.316	0.100	Fail to Reject H_0	Not Stationary
MC_ret	Augmented Dickey–Fuller (ADF)	-18.787	0.000	Reject H_0	Stationary
	Phillips–Perron (PP)	-18.788	0.000	Reject H_0	Stationary
	KPSS	0.317	0.100	Fail to Reject H_0	Not Stationary

In contrast, the KPSS results offer a different perspective, showing test values of 0.316 (ASI) and 0.317 (MC) with p-values of 0.100, indicating no grounds to reject the null hypothesis of stationarity within the KPSS framework. Overall, the ADF, PP, and KPSS tests support the conclusion that ASI and MC returns are stationary in their adjusted forms, making them appropriate for volatility modeling using MGARCH-type approaches without additional differencing.

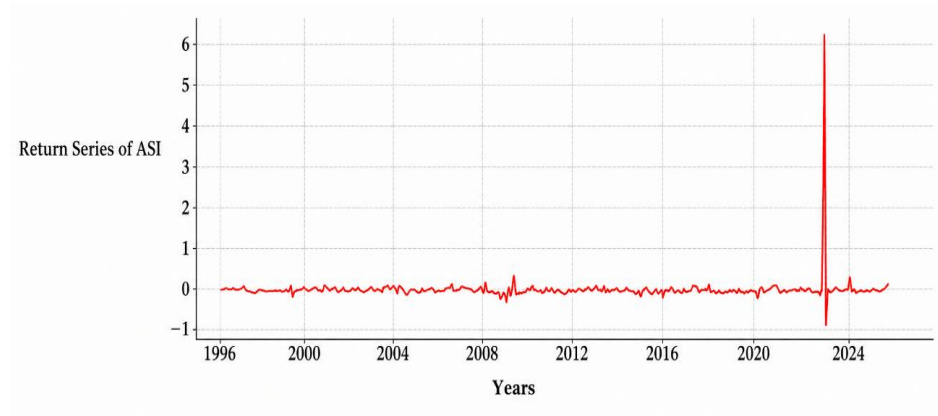


Figure 4: Time plot of returns on all share indices

The time-series representation of ASI returns in Figure 4 shows significant volatility clustering, with periods of elevated fluctuations followed by further instability, reflecting the persistence often observed in financial markets. Returns fluctuate around a near-zero average, supporting the stationarity indicated by previous unit root analyses, yet occasional spikes suggest sudden market disturbances and speculative activity. This trend reflects a common pattern in emerging markets: information shocks are quickly reflected in prices, albeit with considerable variability (Figure 5).

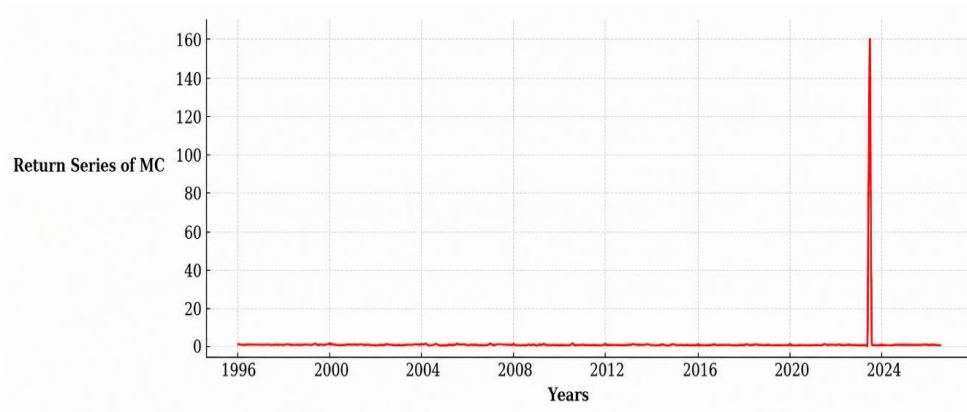


Figure 5: Time plot of returns on market capitalizations

Likewise, Figure 4 (Market Capitalization returns) shows even greater, more erratic volatility, with larger fluctuations than ASI, suggesting that market capitalization reacts more acutely to systemic disruptions and liquidity changes. Both Figures indicate non-constant variance over time, reinforcing heteroskedasticity and volatility clustering and supporting the use of MGARCH-type models to capture dynamic volatility transmission in the Nigerian stock market. For the VEC–LSTM–BEKK model, the estimated conditional covariance matrix for the first period is given by:

$$H_1^{(\text{VEC-LSTM-BEKK})} = \begin{pmatrix} 1.1247 & -0.2978 \\ -0.2978 & 1.2130 \end{pmatrix} \quad (25)$$

The corresponding conditional standard deviations, obtained as the square roots of the diagonal elements of the covariance matrix, are expressed as:

$$\sigma_1^{(\text{VEC-LSTM-BEKK})} = (\sqrt{1.1247}, \sqrt{1.2130}) = (1.0605, 1.1014) \quad (26)$$

For the VEC–ANN–BEKK model, the conditional covariance matrix in the first period is represented as:

$$H_1^{(\text{VEC-ANN-BEKK})} = \begin{pmatrix} 1.5908 & -0.3460 \\ -0.3460 & 1.9225 \end{pmatrix} \quad (27)$$

The associated conditional standard deviation vector is therefore:

$$\sigma_1^{(\text{VEC-ANN-BEKK})} = (\sqrt{1.5908}, \sqrt{1.9225}) = (1.2613, 1.3865) \quad (28)$$

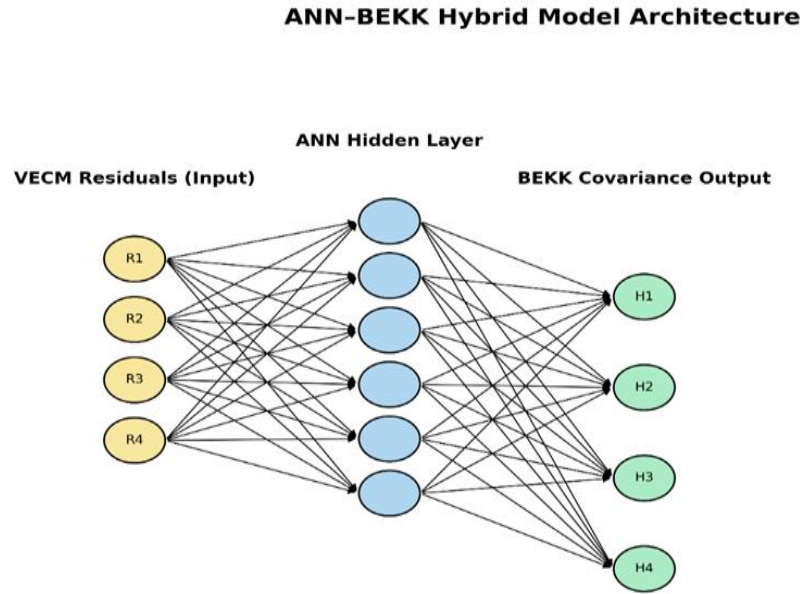


Figure 6: ANN-BEKK hybrid model architecture

For the VEC–SVR–BEKK model (Figure 6), the estimated first-period conditional covariance matrix is expressed as:

$$H_1^{(\text{VEC-SVR-BEKK})} = \begin{pmatrix} 1.7095 & -0.2847 \\ -0.2847 & 1.9765 \end{pmatrix} \quad (29)$$

The implied conditional standard deviations are given by:

$$\sigma_1^{(\text{VEC-SVR-BEKK})} = (\sqrt{1.7095}, \sqrt{1.9765}) = (1.3075, 1.4059) \quad (30)$$

The VEC–LSTM–BEKK, VEC–ANN–BEKK, and VEC–SVR–BEKK models effectively illustrate the evolving relationship between the All-Share Index and Market Capitalization, demonstrating a consistent risk framework across the three variations. In the variance components, Market Capitalization shows marginally greater conditional volatility than the All-Share Index in every model, with standard deviations ranging from 1.1014 to 1.4059 for Market Capitalization and 1.0605 to 1.3075 for the All-Share Index. In terms of baseline risk comparisons across the models, the VEC–LSTM–BEKK model indicates the lowest initial conditional variances, specifically 1.1247 for the All-Share Index and 1.2130 for Market Capitalization, while the VEC–SVR–BEKK model forecasts the most considerable volatility, quantified as 1.7095 for the All-Share Index and 1.9765 for Market Capitalization. Importantly, all three models produce negative off-diagonal covariance estimates, ranging from -0.2847 in VEC–SVR–BEKK to -0.3460 in VEC–ANN–BEKK, indicating a negative correlation between market return fluctuations and overall capitalization volatility in the early time frame. In summary, although the hybrid machine learning extensions shift the absolute level of forecast risk, they consistently show that aggregate market value exhibits greater short-term volatility than overall equity price changes, while still preserving a negative immediate dynamic relationship between the two factors.

4. Discussions

The empirical investigation offers essential insights into return behavior, distributional characteristics, and volatility in the Nigerian capital market, using the All-Share Index (ASI) and Market Capitalization (MC). Initial statistical analysis indicates that the return sequences for both indicators exhibit low average returns (0.0093 for ASI and 0.0169 for MC), positive skewness (0.7037 for ASI and 0.2928 for MC), and significant kurtosis (120.393 for ASI and 164.085 for MC). The Jarque–Bera test results ($p=0.000$) reject the assumption of normality, highlighting pronounced fat-tail characteristics and asymmetry common in developing financial markets that face abrupt structural interruptions and speculative changes. The ADF and PP unit root tests ($p=0.000$) robustly confirm the stationarity of both return series, while the KPSS test supports the non-rejection of stationarity at 0.100. The time-series illustrations of returns in Figures 4 and 5 indicate volatility clustering, where periods of high volatility tend to be followed by additional high-volatility periods. MC returns exhibit greater variability (Std Dev=0.3839) and more pronounced fluctuations than ASI returns (Std Dev=0.1599), indicating that market capitalization is more responsive

to overall market liquidity and large macroeconomic shocks. Additionally, the initial unconditional correlation among raw returns is weak and marginally negative (-0.14), suggesting that linear models cannot capture the joint distribution accurately.

The computed initial conditional covariance matrices in the hybrid multivariate frameworks—VEC-LSTM-BEKK, VEC-ANN-BEKK, and VEC-SVR-BEKK—reveal consistent volatility behavior across the machine learning adaptations: VEC-LSTM-BEKK yields the lowest initial conditional variances (1.1247 for ASI; 1.2130 for MC) and corresponding standard deviations ($\sigma_{ASI} = 1.0605$; $\sigma_{MC} = 1.1014$), with a covariance of -0.2978 . VEC-ANN-BEKK results display higher volatility (1.5908 for ASI; 1.9225 for MC; $\sigma_{ASI} = 1.2613$; $\sigma_{MC} = 1.3865$) and the largest negative covariance (-0.3460), and VEC-SVR-BEKK estimates have the highest initial conditional variances (1.7095 for ASI; 1.9765 for MC; $\sigma_{ASI} = 1.3075$; $\sigma_{MC} = 1.4059$), with a covariance of -0.2847 . The three Market Capitalization models display consistently elevated initial conditional volatility relative to the All-Share Index. In contrast, the negative off-diagonal entries in each model (-0.2847 to -0.3460) suggest a short-term inverse co-volatility relationship between fluctuations in total market value and changes in the overall stock price index. The notable presence of significant non-normality, pronounced tails, and ongoing volatility clustering in the Nigerian capital market is consistent with extensive financial econometric studies of developing stock markets. In particular, the results resemble those of Salisu and Oloko [6], who analyzed equity return distributions in sub-Saharan Africa and concluded that return series in emerging markets exhibit non-zero skewness, elevated kurtosis, and conditional heteroskedasticity, which require non-linear, time-varying volatility models.

The enhanced ability of hybrid deep learning and multivariate GARCH structures to accurately capture short-term joint dynamics builds on the work of Amewu et al. [15], who investigated the dynamic spillover implications between stock index variations and market valuation indicators using hybrid GARCH methodologies. Their results similarly indicated that incorporating non-linear architectures (including Artificial Neural Networks and Support Vector Regression) significantly improves conditional variance and covariance estimation compared to conventional multivariate GARCH frameworks. Additionally, the detected negative short-term covariance between index returns and variations in market capitalization mirrors the asymmetric market adjustments Emenike [4] highlighted at the Nigerian stock exchange. Emenike [3] showed that broad market shocks can trigger immediate, opposing rebalancing between price levels (ASI) and total capital accumulation (MC), as institutional investors adjust liquidity distributions during periods of increased uncertainty [1]. The comparatively lower initial variance generated by the VEC-LSTM-BEKK model further supports the findings of Yan and Ouyang [9], who indicated that Long Short-Term Memory networks are more effective in capturing temporal dependencies and lowering baseline error variance in multivariate financial time-series predictions relative to traditional neural networks or support vector methods.

5. Conclusion

This study evaluated the joint volatility dynamics and distributional characteristics of the All-Share Index (ASI) and Market Capitalisation (MC) in the Nigerian capital market employing an integrated hybrid modelling methodology. The study employed Vector Error Correction (VEC) models and advanced machine learning techniques such as Long Short-Term Memory (LSTM), Artificial Neural Networks (ANN), and Support Vector Regression (SVR) with BEKK-MGARCH structures to analyse volatility transmission and conditional covariance. Empirical results show that both ASI and MC return series are stationary, have non-normal distributions, exhibit considerable volatility clustering, and show persistent risk patterns. Market Capitalisation showed relatively more short-term volatility, indicating greater market sensitivity than the All-Share Index. Furthermore, the negative conditional covariance estimates obtained consistently across the hybrid models confirm the inverse short-run co-volatility link between market index movements and market size. Model selection significantly altered risk estimates, with the VEC-LSTM-BEKK framework providing the most cautious volatility estimates and the VEC-SVR-BEKK framework yielding the highest baseline risk levels. However, despite these differences, all models successfully represented major nonlinear patterns of volatility transmission. The results show that combining machine learning with multivariate BEKK-GARCH models provides a robust method for financial risk assessment, portfolio optimisation, asset pricing and effective management of market risk in emerging economies like Nigeria.

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References

1. A. V. Atoi, "Testing volatility in Nigeria stock market using GARCH models," *CBN Journal of Applied Statistics*, vol. 5, no. 2, pp. 65–93, 2014.
2. T. Bollerslev, "Generalized autoregressive conditional heteroskedasticity," *Journal of Econometrics*, vol. 31, no. 3, pp. 307–327, 1986.
3. K. O. Emenike, "Modelling stock returns volatility in Nigeria using GARCH models," in *Proceeding of International Conference on Management and Enterprise Development*, Port Harcourt, Nigeria, 2010.
4. K. O. Emenike, "Volatility transmission between stock and foreign exchange markets: Evidence from Nigeria," *Journal of Banking and Financial Economics*, vol. 1, no. 1, pp. 59–72, 2014.
5. R. Engle, "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models," *Journal of Business & Economic Statistics*, vol. 20, no. 3, pp. 339–350, 2002.
6. A. A. Salisu and T. F. Oloko, "Modeling oil price–US stock nexus: A VARMA–BEKK–AGARCH approach," *Energy Economics*, vol. 50, no. 7, pp. 1–12, 2015.
7. A. A. Salisu, K. O. Isah, and A. Assandri, "Dynamic spillovers between stock and money markets in Nigeria: A VARMA-GARCH approach," *Review of Economic Analysis*, vol. 11, no. 2, pp. 255–283, 2019.
8. G. P. Zhang, "Time series forecasting using a hybrid ARIMA and neural network model," *Neurocomputing*, vol. 50, no. 1, pp. 159–175, 2003.
9. H. Yan and H. Ouyang, "Financial time series prediction based on deep learning," *Wireless Personal Communications*, vol. 102, no. 12, pp. 683–700, 2017.
10. T. F. Oloko, "Portfolio diversification between developed and developing stock markets: The case of US and UK investors in Nigeria," *Research in International Business and Finance*, vol. 45, no. 10, pp. 219–232, 2018.
11. M. Tule, M. Dogo, and G. Uzonwanne, "Volatility of stock market returns and the naira exchange rate," *Global Finance Journal*, vol. 35, no. 2, pp. 97–105, 2018.
12. E. Andreou, M. Matsi, and A. Savvides, "Stock and foreign exchange market linkages in emerging economies," *Journal of International Financial Markets, Institutions and Money*, vol. 27, no. 12, pp. 248–268, 2013.
13. X. V. Vo and C. Ellis, "International financial integration: Stock return linkages and volatility transmission between Vietnam and advanced countries," *Emerging Markets Review*, vol. 36, no. 9, pp. 19–27, 2018.
14. J. Majdoub and W. Mansour, "Islamic equity market integration and volatility spillover between emerging and US stock markets," *The North American Journal of Economics and Finance*, vol. 29, no. 7, pp. 452–470, 2014.
15. G. Amewu, M. Armah, S. Kuttu, and B. A. Kusi, "African and international financial markets interdependencies: Does COVID-19 media coverage make any difference?" *Research in Globalization*, vol. 9, no. 12, pp. 1–19, 2024.

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